Announcements

1) New Webwork up, due next Thursday

2) Online questionnaire:
put your name in the last question.

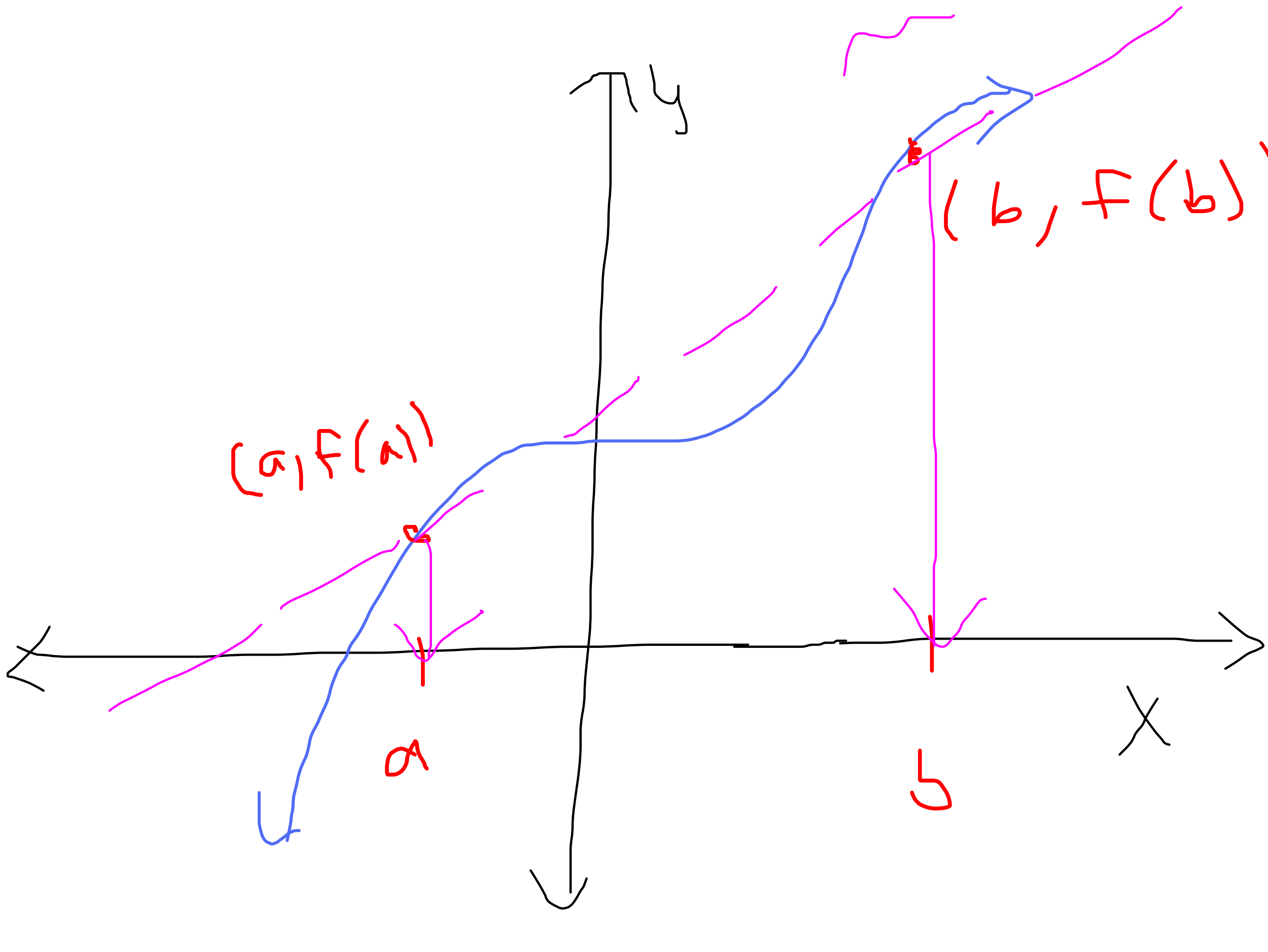
3) Lab Monday

Picture for Mean Value Theorem

Reduce to Rolle's Theorem

$f(a) = f(b)$

slope $\frac{f(b) - f(a)}{b - a}$



Use Mean Value Theorem to show: If $f' > 0$ on some open interval (a, b), then f is increasing on (a,b)

The idea Suppose you're given a and b, $a < b$. By Mean Value Theorem, there is a point c with

$a < c < b$

and $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Since $f'(c) > 0$,

$$0 < f'(c) = \frac{f(b) - f(a)}{b - a}$$

so $0 < \dfrac{f(b) - f(a)}{b-a}$

Multiply by $(b-a)$ on both sides.

$0 < f(b) - f(a)$, so

adding $f(a)$ to both sides,

We get $f(a) < f(b)$,

so f is increasing.

Example 1: Show that

$$p(x) = 5x^7 + 4x^5 + 6x^3 + \pi + x$$

has exactly one zero.

(in the real numbers)

$$p(0) = \pi > 0$$

$$p(-1) = -16 + \pi < 0$$

Since p is continuous on $[-1, 0]$, by the

Intermediate Value Theorem, p has a root in (-1,0).

$$p(x) = 5x^7 + 4x^5 + 6x^3 + x + \pi$$

$$\boxed{p'(x) = 35x^6 + 20x^4 + 18x^2 + 1}$$

Let a = the zero in (-1,0).

Let b = another zero.

By the Mean Value Theorem, since p is differentiable on (a,b), there is a point c with $a < c < b$ and

$$p'(c) = \frac{p(b) - p(a)}{b-a}$$

$$= 0$$

since $p(a) = p(b) = 0$.

so $p'(c) = 35c^6 + 20c^4 + 18c^2 + 1$

$= 0$

But $p'(x) \geq 1$ for all real values of x.

(all powers are even)

This shows that p could not possibly have two zeros, so therefore it has only one.

Optimization Problems

(Section 3.7)

More Story Problems!

Example 2: (from Lutzer)

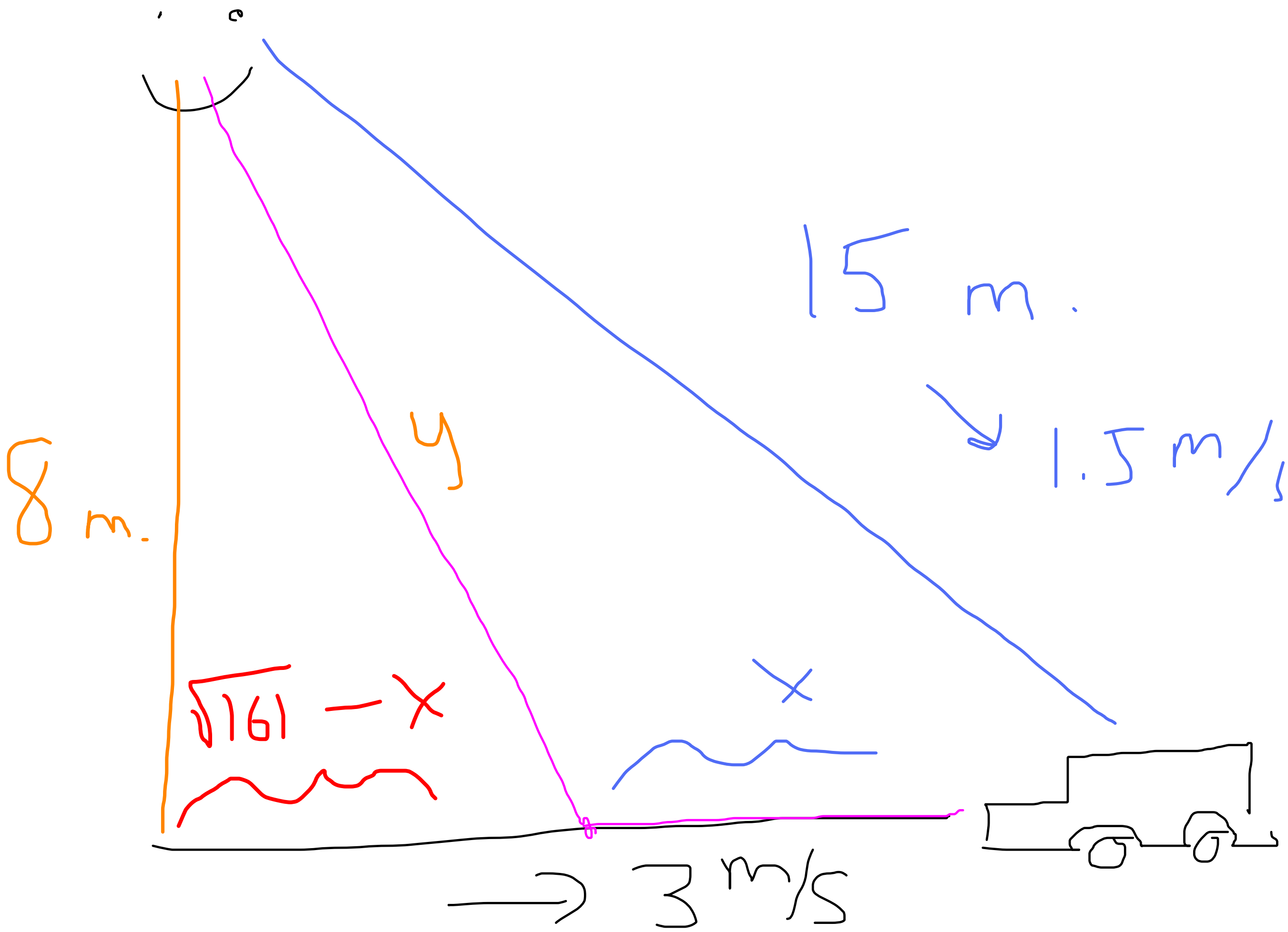
You are on an African safari! You are 15 meters into the savannah on a direct line from your truck. You spot a lion(ness) who has decided that you would make a tasty dinner.

If the horizontal distance between you and your truck is 8 meters, you run at 3 m/s on the road your truck is parked and at 1.5 m/s in the savannah, what route should you take to get to your truck the fastest?

If the lionness runs at a speed of 16 m/s, will you be eaten?

Lionness is 30 m. away.

Draw a picture.



"purple = optimal"

Speed in Savannah = 1.5 m/s

speed on road = 3 m/s

$d = r \cdot t$, so

$t = \frac{d}{r}$ on each

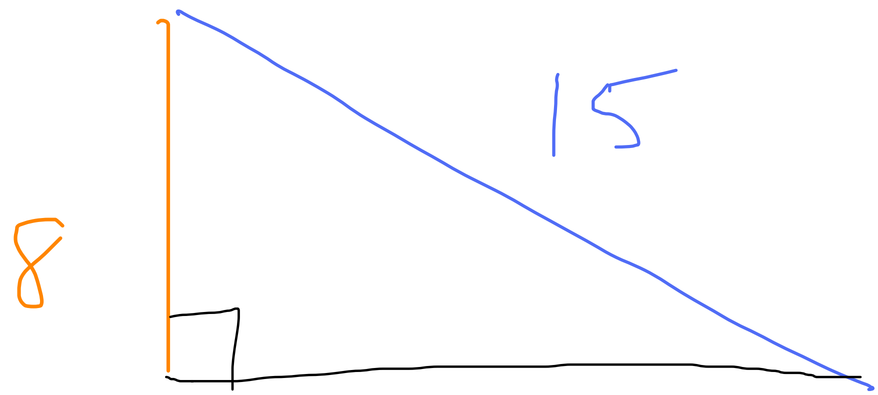
piece (savannah + road)

$$\text{Total time} = \frac{\text{sav. dist}}{\text{sav. rate}} + \frac{\text{road dist}}{\text{road rate}}$$

$$T = \frac{y}{1.5} + \frac{x}{3}$$

Solve for either y in terms of x or x in terms of y.

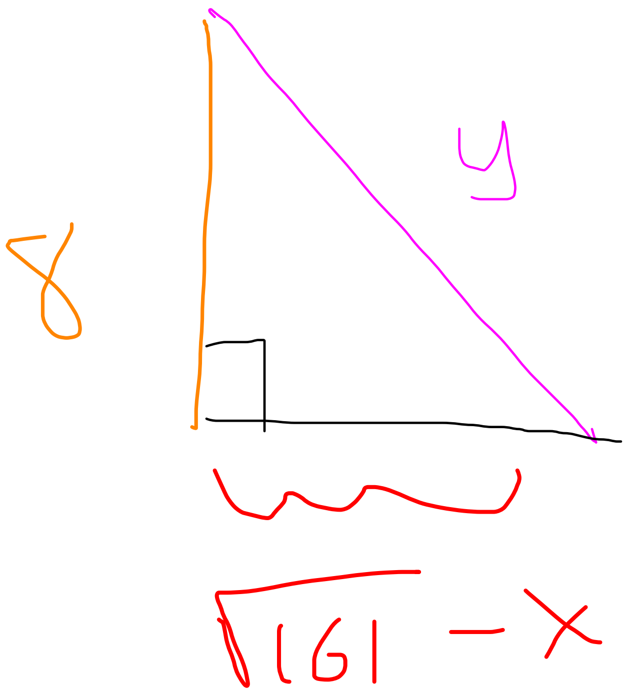
Big triangle



$$\sqrt{15^2 - 8^2}$$

$$= \sqrt{161}$$

Small triangle



$$64 + \left(\sqrt{161} - x\right)^2 = y^2$$

then $y = \sqrt{64 + \left(\sqrt{161} - x\right)^2}$

Then

$$T = \frac{y}{1.5} + \frac{x}{3}$$

$$= \frac{\left((\sqrt{161} - x)^2 + 64\right)^{1/2}}{1.5} + \frac{x}{3}$$

Find the critical points of T. Take derivative with respect to x.

$$\frac{dT}{dx} = \frac{\frac{1}{2}\left(\left(\sqrt{161}-x\right)^2+64\right)^{-1/2}(-2)\left(\sqrt{161}-x\right)}{1.5} + \frac{1}{3} = 0$$

so

$$-\frac{1}{\cancel{3}} = \frac{(-2)\left(\sqrt{161}-x\right)\left(\left(\sqrt{161}-x\right)^2+64\right)^{-1/2}}{\cancel{3}}$$

divide by -2

$$\frac{1}{2} = \left(\sqrt{161}-x\right)\left(\left(\sqrt{161}-x\right)^2+64\right)^{-1/2}$$

Square both sides,

pray that it works.

$$\frac{1}{4} = \left(\sqrt{161}-x\right)^2\left(\left(\sqrt{161}-x\right)^2+64\right)^{-1}$$

$$= \frac{\left(\sqrt{161}-x\right)^2}{\left(\sqrt{161}-x\right)^2+64}$$ cross-mult.

$$\left(\sqrt{161}-x\right)^2+64 = 4\left(\sqrt{161}-x\right)^2$$

square out.

$$x^2 - 2\sqrt{161}x + 161 + 64$$

$$= 4(x^2 - 2\sqrt{161}x + 161)$$

$$= 4x^2 - 8\sqrt{161}x + 644$$

Set equal to zero

$$3x^2 - 6\sqrt{161}x + 419 = 0$$

Use quadratic formula